Precalculus
Functions and Graphs

Seventh Edition

M. A. Munem

J. P. Yizze

Macomb College

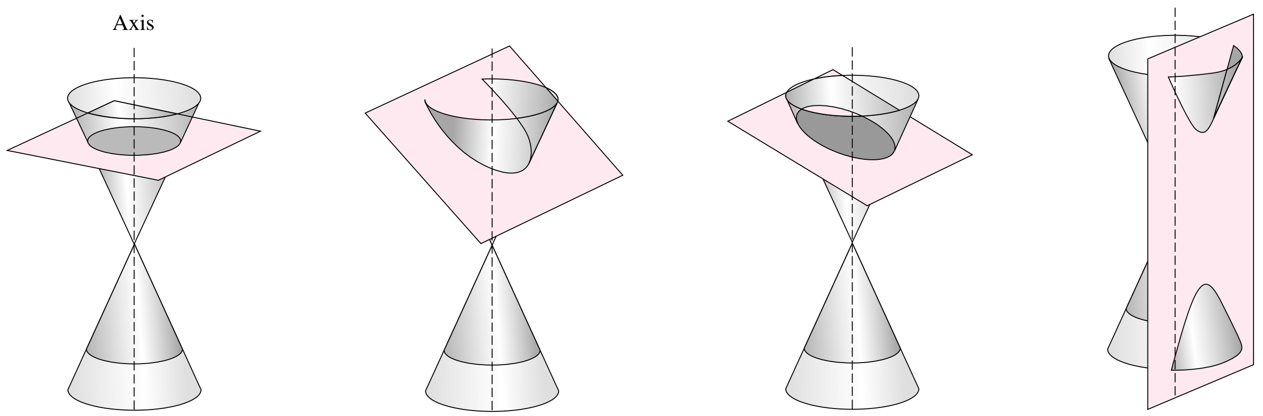
KENDALL/HUNT PUBLISHING COMPANY
4050 Westmark Drive Dubuque, Iowa 52002

9 Topics in Analytic Geometry

CHAPTER CONTENTS

Parabolic shapes appear often in real-world situations. For instance, consider a satellite dish antenna with a parabolic cross section. Suppose the open end of the dish has a diameter of 8 feet and the dish is 1.5 feet deep. Because of the reflecting property of parabolic, the receiver is placed at the focus of the parabolic cross section. Find how far from the vertex the receiver should be placed. The solution to this problem is given in Example 6 on page 621.

Few tools in mathematics have as many varied applications as conic sections. The term *conic sections* (or *conics,* for short) refers to the various figures, or sections, formed by intersecting a right circular cone with a plane. When the plane is perpendicular to the axis of the cone, a *circle* is formed (Figure 1a). When the plane is not perpendicular to the axis of the cone, its intersection forms a *parabola* (Figure 1b), an *ellipse* (Figure 1c), or a *hyperbola* (Figure 1d).



(a) circle (b) parabola (c) ellipse (d) hyperbola

Figure 1

These plane curves are shown in Figure 2.


(a) circle (b) parabola (c) ellipse (d) hyperbola

Figure 2

The connection between geometry and algebra enables us to analyze certain relationships algebraically and interpret certain algebraic relationships geometrically. Our objectives in this chapter center around graphing algebraic equations of useful geometric figures. Earlier, in chapter 1, we discussed circles. Here, we derive standard equations in Cartesian and polar form for parabolas, ellipses, and hyperbolas. We use these curves to illustrate the idea of rotation of axes. We also introduce the general concept of a conic section and then use the ideas of eccentricity and directrix to determine the particular conic. Parametric equations are also included in the chapter.

OBJECTIVES

1. Graph Parabolas with Vertex at (0, 0)
2. Graph Parabolas with Vertex at (*h*, *k*)
3. Solve Applied Problems
4. Explore with Technology

9.1 Parabolas

In Section 3.1 we indicated that the graphs of quadratic functions are parabolas. In this section we give a proof of this fact, based on the following geometric definition:

DEFINITION
PARABOLA

> A **parabola** is the set of all points P in a plane, such that the distance from P to a fixed point (the **focus**) is equal to the distance from P to a fixed line (the **directrix**).

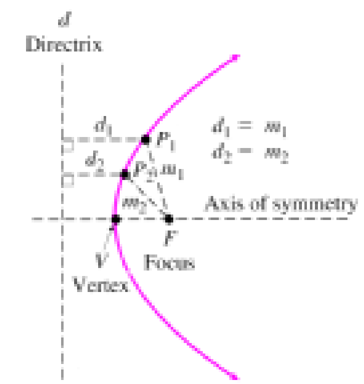


Figure 1

In Figure 1 the points P_1 and P_2 are on the parabola with focus at point F and vertical directrix d to the left of the focus. The distance from P_1 to d, denoted by d_1, is equal to the distance from P_1 to F, denoted by m_1, that is, $d_1 = m_1$. Also, $d_2 = m_2$. The line containing the focus and perpendicular to the directrix is the **axis of symmetry** of the parabola, and the point V where the parabola intersects its axis of symmetry is called the **vertex.**

Graphing Parabolas with Vertex at (0, 0)

To derive the standard equation of a parabola with vertex at (0, 0), we begin by choosing a coordinate system in such a way that the directrix is vertical and the origin is midway between the focus and directrix, as displayed in Figure 2. If the distance between the focus F and the origin is c (where $c > 0$), then the distance from the origin to the directrix is also c. Thus the focus F is located at $(c, 0)$ and the equation of the directrix is $x = -c$.

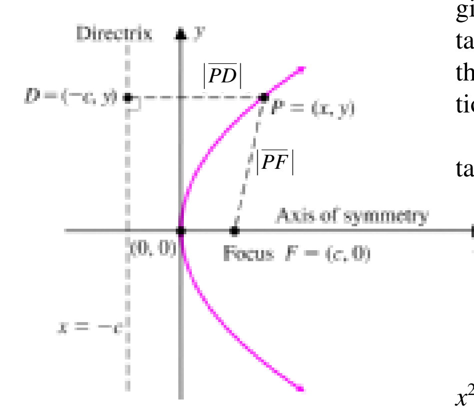


Figure 2

Next we let $P = (x, y)$ represent any point on the parabola. By definition, the distance from P to F is equal to the distance from P to the directrix:

that is,

$$|PF| = |PD|$$

$$\sqrt{(x-c)^2 + y^2} = |x + c| \qquad \text{Distance formula and absolute value property}$$

$$\left.\begin{aligned}(x-c)^2 + y^2 &= (x+c)^2 \\ x^2 - 2cx + c^2 + y^2 &= x^2 + 2cx + c^2\end{aligned}\right\} \qquad \text{Square both sides and multiply}$$

After simplifying the equation, we get

$$y^2 = 4cx \qquad c > 0$$

We refer to this equation (in the box) as the **standard equation** for a parabola with vertex at the origin and a horizontal axis of symmetry. The equation of the directrix is $x = -c$ and the focus is at $(c, 0)$.

Every point (x, y) on the parabola satisfies the equation $y^2 = 4cx$. Conversely, if (x, y) is a point satisfying the equation, then by reversing the steps above, we find that the point (x, y) is on the parabola.